Dr. Laurence Lavelle

Integration Rules

Although integration can be a difficult concept to master, taking integrals doesn't have to be challenging. By following a few simple rules, you'll be able to solve a wide variety of integrals. Here are some common rules of integration that you may find helpful.

Multiplication by a Constant (Used in Chem 14B)
If $f(x) = cg(x)$, then $\int f(x)dx = c\int g(x)dx$ where c is a constant.
Example: $\int 5xdx = 5 * \int xdx = 5 * (\frac{1}{2}x^2 + C)$

The following are two examples of this in Chem 14B.

Work done during a volume expansion against a constant pressure.
$w = -\int_{v1}^{v2} P_{ex}\, dV = -P_{ex}\int_{v1}^{v2} dV = -P_{ex}\,\Delta V$ where P_{ex} is constant
and ΔV = final volume – initial volume = $V_2 - V_1$.

Work done during a volume expansion against a changing pressure.
Here the pressure is substituted out using the ideal gas equation, *PV* = nR*T*, and for an isothermal expansion (T constant) of a closed system (n constant) the terms nRT are all constant and can be moved out of the definite integral, where for easier visual reading $\int_{v1}^{v2}$ is simplified to $\int$.

$$w = -\int P_{ex}\, dV = -\int \frac{nRT}{V} dV = -nRT\int V^{-1} dV = -nRT(lnV_2 - lnV_1) = -nRTln\frac{V_2}{V_1}$$

Power Rule (Used in Chem 14B)

If $f(x) = x^n$, then $\int f(x)dx = \frac{x^{n+1}}{n+1} + C$ where n is a number.

Example: $\int x^3 dx = \frac{x^{3+1}}{3+1} + C = \frac{x^4}{4} + C$

Sum Rule

If $f(x) = g(x) + h(x)$, then $\int f(x)dx = \int g(x)dx + \int h(x)dx$

Example: $\int (x + x^2)dx = \int x dx + \int x^2 dx = \frac{x^2}{2} + \frac{x^3}{3} + C$

Difference Rule

If $f(x) = g(x) - h(x)$, then $\int f(x)dx = \int g(x)dx - \int h(x)dx$

Example: $\int (x - x^2)dx = \int x dx - \int x^2 dx = \frac{x^2}{2} - \frac{x^3}{3} + C$