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Linear, Quadratic, and Cubic Equations With Applications in Chemistry

Linear equations

Slope-intercept form: $y = mx + b$

Represents a straight line with slope *m* and y-intercept *b*

- Math: Write the equation of the line with slope of -3 and a y-intercept of 15.

$$y = -3x + 15$$

- Chemistry: $\ln[A] = -kt + \ln[A]_0$ (First-order integrated rate law)

  - The rate constant for the first order reaction of the decomposition of dinitrogen pentoxide is 0.037/min. A 27.0 gram sample is dissolved in CCl_4 and allowed to decompose. How long will it take to reduce the amount of N_2O_5 to 6.0 grams?

$$N_2O_{5(g)} \rightarrow N_2O_{4(g)} + \frac{1}{2} O_{2(g)}$$

$$\ln[A] = -kt + \ln[A]_0$$

$$\ln[6.0] = -0.037t + \ln[27.0]$$

$$\ln\left[\frac{6.0}{27.0}\right] = -0.037t$$

$$t = 41\ mins$$

Point-slope form: $y - y_1 = m(x - x_1)$

Represents a straight line with slope *m* that connects the points (x_1, y_1) and (x_2, y_2)

- Math: Write the equation of the line that goes through the points (3, 4) and (5, 8)

$$m = \frac{\Delta y}{\Delta x} = \frac{8-4}{5-3} = 2$$

$$y - 4 = 2(x - 3)$$

- Chemistry: $\ln k_1 - \ln k_2 = \frac{E_a}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$

  - The activation energy for a first order reaction is 1500. J/mol. The rate constant was experimentally found to be 0.20 s^{-1} at 500. K. Calculate the rate constant at 200. K.

$$\ln k_1 - \ln k_2 = \frac{E_a}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$\ln 0.20 - \ln k_2 = \frac{1500\frac{J}{mol}}{8.314\frac{J}{mol.K}}\left(\frac{1}{200K} - \frac{1}{500K}\right)$$

$$\ln 0.20 - \ln k_2 = 0.541$$

$$-2.15 = \ln k_2$$

$$k_2 = 0.12 \text{ s}^{-1}$$

Quadratic equations

$$ax^2 + bx + c = 0$$

The solutions are given by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Math: Find the two solutions for the equation, $x^2 + 3x - 10 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 + 40}}{2}$$

$$x = \frac{-3 \pm \sqrt{49}}{2}$$

$$x = \frac{-3 \pm 7}{2}$$

$$x = 2\ and\ -5$$

- Chemistry: 0.75 mol PCl_5 is placed in a 500ml reaction vessel and decomposes at 250^{0}C to form PCl_3 and Cl_2. K_c = 1.80, All 3 substances are gases at 250^{0}C.

$$PCl_5 \rightleftharpoons PCl_3 + Cl_2$$

	PCl_5	PCl_3	Cl_2
Initial	1.50	0	0
Change	- X	+ X	+ X
Equilibrium	1.50 - X	X	X

$$K = \frac{[PCl_3][Cl_2]}{[PCl_5]} = \frac{x^2}{1.50 - X} = 1.80$$

$$x^2 + 1.80x - 2.7 = 0$$

$$x = \frac{-1.80 \pm \sqrt{1.80^2 - 4(1)(-2.7)}}{2(1)}$$

$$x = \frac{-1.80 \pm \sqrt{3.24 + 10.8}}{2}$$

$$x = -2.77 \; and \; 0.973$$

Since concentrations must be positive, X = 0.973 M
The composition of the equilibrium mixture is:
$[PCl_5]$ = 1.50 – X = 0.53 M
$[PCl_3]$ = X = 0.97 M
$[Cl_2]$ = X = 0.97M

*In chemistry only one of the two solutions for X in the quadratic equation will be valid. One value will either be negative or it will be too big, such that when you subtract it from the initial concentration, you get a negative concentration which is not reasonable.

Cubic equations

$$ax^3 + bx^2 + cx + d = 0$$

The explicit formula for the solutions (analogous to the quadratic formula) is exceedingly complicated for cubic equations.

To avoid these cubic equations, in Chem 14A you will assume that X is small enough to be negligible, eg. 0.20 – X ≈ 0.20. This assumption is valid when the equilibrium constant is small (typically $K < 10^{-4}$).

- Chemistry: What is the pH of a 0.20M NH_4Cl salt solution? $K_a = 5.6 \times 10^{-10}$

$$NH_4^+{}_{(aq)} + H_2O_{(l)} \rightleftharpoons H_3O^+{}_{(aq)} + NH_{3(aq)}$$

	NH_4^+	H_2O	H_3O^+	NH_3
Initial	0.20	-	0	0
Change	- X	-	+ X	+ X
Equilibrium	0.20 - X	-	X	X

$$K = \frac{[H_3O^+][NH_3]}{[NH_4{}^+]} = \frac{x^2}{0.20 - X} = 5.6\,x\,10^{-10}$$

Because $5.6 \times 10^{-10} < 10^{-4}$, we can assume X to be negligible.

Therefore the denominator simplifies to only 0.20.

$$\frac{x^2}{0.20} = 5.6\,x\,10^{-10}$$

$$x = 1.1\,x\,10^{-5}$$

$$pH = -\log\,[1.1\,x\,10^{-5}]$$

$$pH = 4.98$$