

Linear, Quadratic, and Cubic Equations With Applications in Chemistry

Linear equations

Slope-intercept form: $y = mx + b$

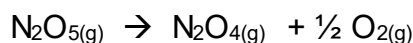
Represents a straight line with slope m and y-intercept b

- Math: Write the equation of the line with slope of -3 and a y-intercept of 15.

$$y = -3x + 15$$

- Chemistry: $\ln[A] = -kt + \ln[A]_0$ (First-order integrated rate law)

- The rate constant for the first order reaction of the decomposition of dinitrogen pentoxide is 0.037/min. A 27.0 gram sample is dissolved in CCl_4 and allowed to decompose. How long will it take to reduce the amount of N_2O_5 to 6.0 grams?



$$\ln[A] = -kt + \ln[A]_0$$

$$\ln[6.0] = -0.037t + \ln[27.0]$$

$$\ln\left[\frac{6.0}{27.0}\right] = -0.037t$$

$$t = 41 \text{ mins}$$

Point-slope form: $y - y_1 = m(x - x_1)$

Represents a straight line with slope m that connects the points (x_1, y_1) and (x_2, y_2)

- Math: Write the equation of the line that goes through the points (3, 4) and (5, 8)

$$m = \frac{\Delta y}{\Delta x} = \frac{8 - 4}{5 - 3} = 2$$

$$y - 4 = 2(x - 3)$$

- Chemistry: $\ln k_1 - \ln k_2 = \frac{E_a}{R} \left(\frac{1}{T_2} - \frac{1}{T_1} \right)$

- The activation energy for a first order reaction is 1500. J/mol. The rate constant was experimentally found to be 0.20 s⁻¹ at 500. K. Calculate the rate constant at 200. K.

$$\ln k_1 - \ln k_2 = \frac{E_a}{R} \left(\frac{1}{T_2} - \frac{1}{T_1} \right)$$

$$\ln 0.20 - \ln k_2 = \frac{1500 \frac{J}{mol}}{8.314 \frac{J}{mol \cdot K}} \left(\frac{1}{200K} - \frac{1}{500K} \right)$$

$$\ln 0.20 - \ln k_2 = 0.541$$

$$-2.15 = \ln k_2$$

$$k_2 = 0.12 \text{ s}^{-1}$$

Quadratic equations

$$ax^2 + bx + c = 0$$

The solutions are given by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Math: Find the two solutions for the equation, $x^2 + 3x - 10 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 + 40}}{2}$$

$$x = \frac{-3 \pm \sqrt{49}}{2}$$

$$x = \frac{-3 \pm 7}{2}$$

$$x = 2 \text{ and } -5$$

- Chemistry: 0.75 mol PCl_5 is placed in a 500ml reaction vessel and decomposes at 250°C to form PCl_3 and Cl_2 . $K_c = 1.80$, All 3 substances are gases at 250°C .



	PCl_5	PCl_3	Cl_2
Initial	1.50	0	0
Change	- X	+ X	+ X
Equilibrium	1.50 - X	X	X

$$K = \frac{[\text{PCl}_3][\text{Cl}_2]}{[\text{PCl}_5]} = \frac{x^2}{1.50 - X} = 1.80$$

$$x^2 + 1.80x - 2.7 = 0$$

$$x = \frac{-1.80 \pm \sqrt{1.80^2 - 4(1)(-2.7)}}{2(1)}$$

$$x = \frac{-1.80 \pm \sqrt{3.24 + 10.8}}{2}$$

$$x = -2.77 \text{ and } 0.973$$

Since concentrations must be positive, $X = 0.973 \text{ M}$

The composition of the equilibrium mixture is:

$$[\text{PCl}_5] = 1.50 - X = 0.53 \text{ M}$$

$$[\text{PCl}_3] = X = 0.97 \text{ M}$$

$$[\text{Cl}_2] = X = 0.97 \text{ M}$$

*In chemistry only one of the two solutions for X in the quadratic equation will be valid. One value will either be negative or it will be too big, such that when you subtract it from the initial concentration, you get a negative concentration which is not reasonable.

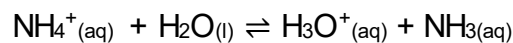
Cubic equations

$$ax^3 + bx^2 + cx + d = 0$$

The explicit formula for the solutions (analogous to the quadratic formula) is exceedingly complicated for cubic equations.

To avoid these cubic equations, in Chem 14A you will assume that X is small enough to be negligible, eg. $0.20 - X \approx 0.20$. This assumption is valid when the equilibrium constant is small (typically $K < 10^{-4}$).

- Chemistry: What is the pH of a 0.20M NH_4Cl salt solution? $K_a = 5.6 \times 10^{-10}$



	NH_4^+	H_2O	H_3O^+	NH_3
Initial	0.20	-	0	0
Change	- X	-	+ X	+ X
Equilibrium	$0.20 - X$	-	X	X

$$K = \frac{[\text{H}_3\text{O}^+][\text{NH}_3]}{[\text{NH}_4^+]} = \frac{x^2}{0.20 - X} = 5.6 \times 10^{-10}$$

Because $5.6 \times 10^{-10} < 10^{-4}$, we can assume X to be negligible.

Therefore the denominator simplifies to only 0.20.

$$\frac{x^2}{0.20} = 5.6 \times 10^{-10}$$

$$x = 1.1 \times 10^{-5}$$

$$\text{pH} = -\log [1.1 \times 10^{-5}]$$

$$\text{pH} = 4.98$$