Everything You Ever Wanted to Learn about Significant Figures and Then Some

Introduction

Ah, significant figures, the bane of many a general chemistry student. These digits, innocuously nicknamed "sig figs," are generally met with aversion bordering on hatred. Why? Because many students find the rules governing sig figs and their use complicated, tedious, and seemingly arbitrary.

Do not allow yourself to be blinded by your contempt! Sig figs are actually quite important and productive members of the scientific community, providing a deliberate and direct way of conveying the number of digits reliably known in a measurement or calculation.

So read on, and get to know sig figs a little better; you may just find you actually like them! (At the very least you'll understand them better.)

Exact vs. Inexact Numbers

Some numbers are exact; they have absolutely no uncertainty associated with them. Exact numbers are always either counted or defined. For example, there are thirty-one days in the month of January. The Imperial unit of a foot is defined as containing exactly twelve inches. Thirty-one days in January. Twelve inches in a foot. These are exact numbers.

Any measured quantity, however, is inexact. Measurements contain inherent limitations that come from both the equipment used to make the measurement and the individual taking the measurement.

Thus arises the need for significant figures!

What are Sig Figs?

Significant figures relay the number of digits reliably known in a number. Measured quantities are typically reported such that only the last significant digit is uncertain.

There are a few important rules for determining which digits in a given number are significant:

1. Non-zero numbers are ALWAYS significant.
2. Zeros between non-zero numbers are ALWAYS significant.

3. Zeros to the left of the first non-zero number are NOT significant. (0.003 has one sig fig. The zeros simply indicate the position of the decimal place)
4. Zeros at the end of a number and to the right of the decimal point are ALWAYS significant.
5. Zeros at the end of a number and to the left of the decimal point are NOT NECESSARILY significant.

The first two rules are fairly straightforward. The next two are a bit more abstract but will become clear with the provided practice problems.

The fifth rule, however, is problematic. What exactly does it mean for a zero to be "not necessarily" significant? So, sometimes it is, and sometimes it isn't? Isn't that a bit ambiguous?

Why yes, yes it is ambiguous—but, fortunately, we have ways to avoid this ambiguity.

If, for instance, the number in question is "6000," and you see the number written 6000.00, you can be confidant that all five zeros are significant, giving the number six sig figs overall. The two zeros to the right of the decimal place must be significant by rule 4.; therefore the zeros before the decimal place must be significant as well.

If the number is written $6000.$ with the decimal place explicitly shown, then it is generally safe to assume that all three zeros are meant to be significant, giving the number four sig figs overall.

But what if we want to express the number 6000 with only three sig figs? Or one? What is the best way write the number? Well, occasionally you might see the number written with a bar over the last significant digit (for instance $60\bar{0}0$ generally means the number has three significant figures). However, this is neither the most common nor the clearest way to express this. The best way to avoid sig fig ambiguity is to use scientific notation!

In scientific notation, the difference between $6.00 * 10^3$ (three sig figs), $6.0 * 10^3$ (two sig figs), and $6 * 10^3$ (one sig fig) is completely unambiguous. As a general rule of thumb, always use scientific notation when determining significant figures or reporting results, especially if the number ends with zeros that are to the left of the decimal place. The most reliable way to represent a number is always with scientific notation.

Rules for Significant Figures in Calculations

Now that we know what significant figures are and how to determine the number of them in a given value, let's learn how to use significant figures in calculations.

Let's say we want to add multiple measurements together, perhaps to take an average, or we need to multiply some numbers to convert units or simply to get to our final answer. How can we determine how many significant digits we ought to report in our final answer?

We will need to use the number of sig figs in the initial values to figure out the sig figs for our final answer. The general rules for this are as follow:

1. Addition and Subtraction

   Number of decimal places in answer = Smallest number of decimal places in the initial data

   Example: $2.0158\ \frac{g}{mol} + 16.00\ \frac{g}{mol} = 18.0158\ \frac{g}{mol} = \mathbf{18.02\ \frac{g}{mol}}$ because the smallest number of decimal places in the given data is two (from the value $16.00\ \frac{g}{mol}$)

2. Multiplication and Division

   Number of significant figures in answer = Smallest number of significant figures in the data

   Example: $18.0158\ \frac{g}{mol} * 4.0\ mol = 72.0632\ g = \mathbf{72\ g}$ because the smallest number of sig figs in the given data is two (from $4.0\ mol$).

3. Rules for Logarithms and Exponentials

   In general chemistry, logarithms and exponentials are encountered far less frequently than addition/subtraction and multiplication/division. Nevertheless, it is important to know the rules for dealing with sig figs for logarithms and exponentials because there will be times you will encounter them (particularly for concepts like pH). The general rules work like this:

When taking the logarithm of a number, your answer should have the same number of decimal places as there are sig figs in the initial number.

Example: $\log(134) = \mathbf{2.127}$ because there are three significant figures in the number we are taking the log of, so there are three decimal places in the answer.

Conversely, when raising to a power, your answer should have the same number of sig figs as decimal places in the power.

Example: $10^{1.558} = \mathbf{36.1}$ because there are three decimal places in the power and therefore three significant figures in the result.

4. Multiple-Step Calculations

When your calculation involves more than one operation, you will need to follow the standard order of operations. You may have learned the order of operations as PEMDAS (parentheses, then exponents, then multiplication/division, and finally addition/subtraction). You should only round your final answer (see the caution below in the next section), but you will need to keep track of the sig figs carried through each step of the calculation if you want to get to the correct answer.

Example: $1.35 + 5.11 * 0.1025$

Step 1) Multiplication: $5.11 * 0.1025 = 0.523775$ (This value carries three sig figs from the initial value with the lowest number of sig figs, 5.11).

Step 2) Addition: $1.35 + 0.523775 = 1.873775$ (This value carries two decimal places from the value with the lowest number of decimal places, 1.35).

Step 3) Round (see next section for rules on rounding): 1.873775 to two decimal places is $\mathbf{1.87}$.

5. Rules for Rounding Your Final Answer

If the last digit is above 5, round up (3.88 to two sig figs is 3.9)

If the last digit is below 5, round down (2.72 to two sig figs is 2.7)

If the last digit is equal to 5, round to the nearest even number

(2.75 to two sig figs is 2.8 but 2.65 to two sig figs in 2.6)

This last rule, known as the "rule of 5," prevents rounding bias because half the time the rounding will overestimate, and half the time the rounding will underestimate. Therefore, over many measurements and calculations, any rounding errors will be averaged out. You can also think of this rule as the following: if the number before the 5 is odd, round up, and if the number before the 5 is even, leave it as it is.

Caution: Round your final answer to the correct number of significant figures. However, during calculations, make sure to carry all of your digits (do not round until the final answer).

For instance, if I want to solve $0.5 * 2.873 + 3.42$, the correct process would be to first evaluate $0.5 * 2.873 = 1.4365$, noting that this result carries one significant figure (in other words, if this were the final result, we would round to 1). Then, evaluate $1.4365 + 3.42 = 4.8565$, noting that this, the final result, should have zero decimal places (because the value plugged in as 1.4365 actually carries zero decimal places). Therefore, our answer would be $\mathbf{5}$. However, if I had incorrectly plugged in the rounded value after the first step, then I would get a final answer of $1 + 3.42 = 4.42 = 4$**.** Avoid these rounding errors by keeping track of the digits throughout the calculation, but rounding only the final answer.